Heegaard Splittings of ∂-reducible 3-manifolds

Jiming Ma* and Ruifeng Qiu*

February 1, 2008

Abstract. In this paper, we shall prove that any Heegaard splitting of a ∂-reducible 3-manifold M, say $M = W \cup V$, can be obtained by doing connected sums, boundary connected sums and self-boundary connected sums from Heegaard splittings of n manifolds $M_1, \ldots, M_n$ where M_i is either a solid torus or a ∂-irreducible manifold. Furthermore, $W \cup V$ is stabilized if and only if one of the factors is stabilized.

Keywords. Connected sum, boundary connected sum, self-boundary connected sum.

AMS Subject Classification: 57N10, 57M50.

§1 Introduction

Let M be a compact 3-manifold with boundary such that each component of ∂M is not a 2-sphere. If there is a 2-sphere in M which does not bound any 3-ball, then we say M is reducible; otherwise, M is irreducible. If there is an essential disk D in M, then we say M is ∂-reducible; otherwise, M is ∂-irreducible.

Let M be a compact 3-manifold. If there is a closed surface S which separates M into two compression bodies W and V with $\partial_+ W = \partial_+ V = S$, then we say M has a Heegaard splittings, denoted by $M = W \cup_S V$ or $M = W \cup V$. In this case, S is called a Heegaard surface of M. A Heegaard splitting $M = W \cup_S V$ is said to be reducible if there are two essential disks $D_1 \subset W$ and $D_2 \subset V$ such that $\partial D_1 = \partial D_2$. A Heegaard splitting

*Both authors are supported by a grant (No.10171038) of NSFC

$M = W \cup_S V$ is said to be ∂-reducible if there is an essential disk D in M such that D intersects S in only one essential simple closed curve. A Heegaard splitting $M = W \cup_S V$ is said to be weakly reducible if there are two essential disks $D_1 \subset W$ and $D_2 \subset V$ such that $\partial D_1 \cap \partial D_2 = \emptyset$. A Heegaard splitting $M = W \cup_S V$ is said to be stabilized if there are two properly embedded disks $D_1 \subset W$ and $D_2 \subset V$ such that D_1 intersects D_2 in only one point. It is easy to see that if $M = W \cup_S V$ is stabilized and $g(S) \geq 2$ then it is reducible.

Now there are some results on reducibilities of Heegaard splittings. For example, Haken proved that any Heegaard splitting of a reducible 3-manifold is reducible; Casson and Gordon gave a disk version of Haken's lemma, that say, any Heegaard splitting of a ∂-reducible 3-manifold is ∂-reducible, they also show that if M has a weakly reducible Heegaard splitting $W \cup V$ then either $W \cup V$ is reducible or M contains an essential closed surface of genus at least one; Ruifeng Qiu recently proved Gordon's conjecture on stabilizations of reducible Heegaard splittings, that say, the connected sum of two Heegaard splittings is stabilized if and only if one of the two factors is stabilized.

In this paper, we shall consider Heegaard splittings of ∂-reducible manifolds. The main result is the following:

Theorem 1. Any Heegaard splitting of a ∂-reducible manifold M, say $M = W \cup V$, can be obtained by doing connected sums, boundary connected sums and self-boundary connected sums from Heegaard splittings of n manifolds $M_1, \ldots, M_n$, where M_i is either a solid torus or an irreducible, ∂-irreducible manifold. Furthermore, $W \cup V$ is stabilized if and only if one of the factors is stabilized.

Remark. If $M = W \cup V$ can be obtained by doing connected sums, boundary connected sums and self-boundary connected sums from Heegaard splittings of l manifolds $X_1, \ldots, X_l$ where X_j is either a solid torus or an irreducible, ∂-irreducible manifold, then $n = l$ and $\{M_1, \ldots, M_n\} = \{X_1, \ldots, X_n\}$. We omit the proof.

A Heegaard splitting of a handlebody H, say $W \cup V$, is said to be trivial if W is homeomorphic to $\partial H \times I$ and V is homeomorphic to H.

As an application, we shall give a new proof to Scharlemann-Thomston's result:

Corollary 2([ST]). Any unstabilized Heegaard splitting of a handlebody is trivial.

§2 Prelimary

Connected sums of Heegaard splittings

Now let $M = W \cup V$ be a reducible Heegaard splitting. Then there is a 2-sphere P such that $B_W = P \cap W$ is an essential disk in W and $B_V = P \cap V$ is an essential disk in V. Suppose that P separates M into M_1^* and M_2^*. Then B_W separates W into W_1^* and W_2^*, B_V separates V into V_1 and V_2. We may assume that $W_1^*, V_1 \subset M_1^*$ and $W_2^*, V_2 \subset M_2^*$. Let $M_1 = M_1^* \cup_P H_1^3$ and $M_2 = M_2^* \cup_P H_2^3$ where H_1^3 and H_2^3 are two 3-balls. Then M is called the connected sum of M_1 and M_2, denoted by $M = M_1 \sharp M_2$. Let $W_1 = W_1^* \cup H_1^3$, and $W_2 = W_2^* \cup H_2^3$. Then W_1 and W_2 are two compression bodies such that $\partial_+ V_1 = \partial_+ W_1$ and $\partial_+ V_2 = \partial_+ W_2$. Hence $M_1 = W_1 \cup V_1$ is a Heegaard splitting of M_1 and $M_2 = W_2 \cup V_2$ is a Heegaard splitting of M_2. In this case, $W \cup V$ is called the connected sum of $W_1 \cup V_1$ and $W_2 \cup V_2$.

Boundary connected sums of Heegaard splittings

Let M be a compact orientable ∂-reducible 3-manifold, and D be an essential disk in M. Suppose that D is separating in M. Then ∂D is also separating in ∂M. Now each component of $M - D \times (0,1)$ is a 3-manifold with boundary, denoted by M_i. Without loss of generality, we may assume that $D \times \{0\} \subset \partial M_1$ and $D \times \{1\} \subset \partial M_2$. In this case, we say M is the boundary connected sum of M_1 and M_2, denoted by $M = M_1 \cup_D M_2$.

Suppose that $M_i = W^i \cup V^i$ is a Heegaard of M_i such that $D \times \{0\} \subset \partial_- V^1$ and $D \times \{1\} \subset \partial_- V^2$. Then there are unknotted, properly embedded arcs α_i in V^i and β in $D \times [0,1]$ such that $\partial_1 \alpha_1 \subset \partial_+ V^1$, $\partial_2 \alpha_1 = \partial_1 \beta$, $\partial_2 \beta = \partial_1 \alpha_2$, $\partial_2 \alpha_2 \subset \partial_+ V^2$. Then $\gamma = \alpha_1 \cup \beta \cup \alpha_2$ is a properly embedded arc in $V^1 \cup D \times [0,1] \cup V^2$. Now let $N(\gamma)$ be a regular neighborhood γ in $V^1 \cup D \times [0,1] \cup V^2$ such that $N(\partial_1 \gamma) \subset \partial_+ V^1$, and $N(\partial_2 \gamma) \subset \partial_+ V^2$. It is easy to see that $W^1 \cup N(\gamma) \cup W^2)$, denoted by W, is a compression body in M, and the closure of $V^1 \cup V^2 - N(\gamma)$, denoted by V, is also a compression body in M. Hence $W \cup V$ is a Heegaard splitting of M. We say $W \cup V$ is the boundary connected sum of the two

Heegaard splittings $W^1 \cup V^1$ and $W^2 \cup V^2$.

Self-boundary connected sums of Heegaard splittings

Let M be a ∂-reducible 3-manifold, and D be an essential disk in M. Suppose that D is non-separating in M, but ∂D is separating in ∂M. Now $M' = M - D \times (0, 1)$ is a connected manifold such that $\partial M'$ contains at least two components F_1 and F_2. We may assume that $D \times \{0\} \subset F_1$ and $D \times \{1\} \subset F_2$. In this case, we say that M is a self-boundary connected sum of M'.

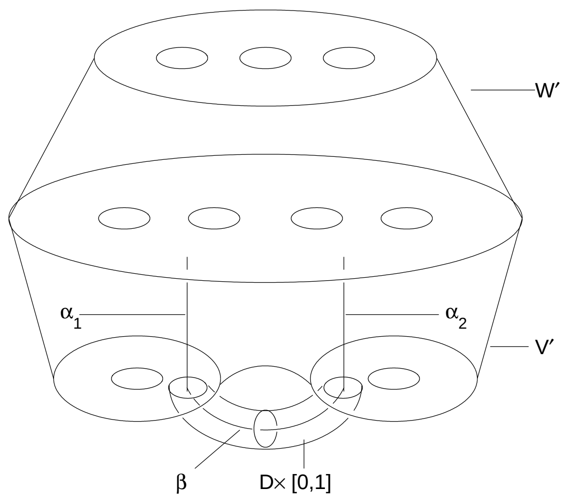

Figure 1

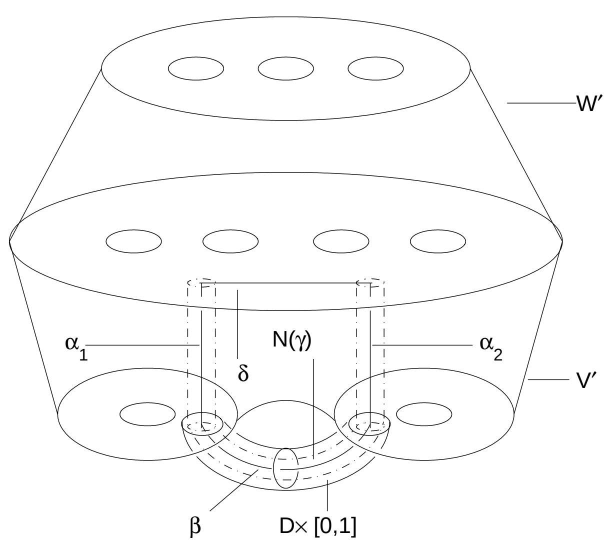

Figure 2

let $M' = W' \cup V'$ be a Heegaard splitting of M', such that $F_1, F_2 \subset \partial_- V'$. Now suppose that α_1, α_2 are two unknotted properly embedded arc in V' such that $\partial_1\alpha_1$ and $\partial_2\alpha_2$ lie in $\partial_+ V'$, and β is a unknotted properly embedded arc in $D \times [0,1]$ such that $\partial_2\alpha_1 = \partial_1\beta$ and $\partial_1\alpha_2 = \partial_2\beta$. Then $\gamma = \alpha_1 \cup \beta \cup \alpha_2$ is a properly embedded arc in $V' \cup D \times [0,1]$. Let $N(\gamma)$ be a regulalr neighborhood of γ in $V' \cup D \times [0,1]$. It is easy to see that $W = W' \cup N(\gamma)$ is a compression body and the closure of $V' \cup D \times [0,1] - N(\gamma)$, denoted by V, is also a compression body. Hence $M = W \cup V$ is a Heegaard splitting of M. We say $W \cup V$ is a self-boundary connected sum of $W' \cup V'$. See Figure 1 and Figure 2.

By definitions, if $W \cup_S V$ is the connected sum or the boundary connected sum of $W^1 \cup_{S_1} V^1$ and $W^2 \cup_{S_2} V^2$, then $g(S) = g(S_1) + g(S_2)$; if $W \cup_S V$ is the self-boundary connected sum of $W' \cup_{S'} V'$, then $g(S) = g(S') + 1$.

§3 Proofs of Theorem 1

Lemma 3.1 ([Q]). The connected sum of two Heegaard splittings is stabilized if and only if one of the two factors is stabilized.

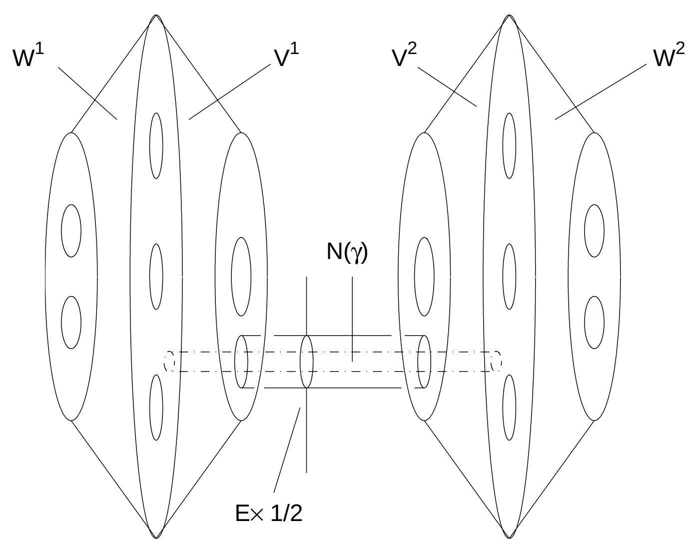

Figure 3

Lemma 3.2. The boundary connected sum of two Heegaard splittings is stabilized if and only if one of the two factors is stabilized.

Proof. Suppose that M_1 and M_2 are two 3-manifolds with boundary, and $M = M_1 \cup D \times [0,1] \cup M_2$ such that $D \times \{0\} \subset \partial M_1$ and $D \times \{1\} \subset \partial M_2$. Suppose that $M_i = W^i \cup V^i$ is a Heegaard splitting of M_i for $i = 1, 2$, and $M = W \cup_S V$ is the boundary connected sum of $W^1 \cup V^1$ and $W^2 \cup V^2$ as in Figure 3.

Assume first that one of $W^1 \cup V^1$ and $W^2 \cup V^2$, say $W^1 \cup V^1$, is stabilized. Then there are two essential disks $B_W \subset W^1$ and $B_V \subset V^1$ such that $|B_W \cap B_V| = 1$. It is easy to see that B_W and B_V can be chosen to be disjoint from $N(\alpha_1) \subset N(\gamma)$. Hence $W \cup_S V$ is also stabilized.

Assume now that $W^1 \cup V^1$ and $W^2 \cup V^2$ are unstabilized. We attach a 2-handle $E \times I$ to M such that $\partial E \times I = \partial D \times I$, where E is a disk. We debote the resulting manifold by M^*. Let $V^* = V \cup E \times I$, then V^* is a compression body. Hence $M^* = M \cup E \times I$ has a Heegaard splitting $M^* = W \cup_S V^*$. Since $D \times \{t\}$ intersects S in an essential simple closed curve which lies in $\partial N(\gamma)$, $P = D \times \{1/2\} \cup E \times \{1/2\}$ is a separating 2-sphere in M which intersects S in an essential simple closed curve. By definition, M^* is just the connected sum of M_1 and M_2 and $M^* = W \cup V^*$ is the connected sum of $W^1 \cup V^1$ and $W^2 \cup V^2$. Since $W^i \cup V^i$ is unstabilized, by Lemma 3.1, $W \cup V^*$ is unstabilized. Hence $W \cup V$ is unstabilized. Q.E.D.

Lemma 3.3. The self-boundary connected sum of a Heegaard splitting $M' = W' \cup V'$ is stabilized if and only if $M' = W' \cup V'$ is stabilized.

Proof. Suppose that $M' = W' \cup V'$ is a Heegaard splitting of M' and $M = W \cup_S V$ is a self-boundary connected sum of $M' = W' \cup V'$ defined in Section 2.

Assume first that $W' \cup V'$ is stabilized. Then there are two essential disks $B_W \subset W'$ and $B_V \subset V'$ such that $|B_W \cap B_V| = 1$. It is easy to see that B_W and B_V can be chosen to be disjoint from $N(\alpha_1 \cup \alpha_2) \subset N(\gamma)$. Hence $W \cup_S V$ is also stabilized.

Assume now that $W' \cup V'$ is unstabilized. We attach a 2-handle $E \times I$ to V such that $\partial E \times I = \partial D \times I$, where E is a disk. Let $M^* = M \cup E \times I$. Then $P = D \times \{1/2\} \cup E \times \{1/2\}$ is a non-separating 2-sphere in M^*. By definition, P intersects S in an essential simple closed curve.

Let δ be an arc in $\partial_+ W'$, such that $\partial\delta = \partial\gamma$ as in Figure 2, then $\gamma \cup \delta$ is a simple closed curve which intersects P in one point. Let $P \times I$ be a regular neighborhood of P in $E \times I \cup D \times I$. Let $a = \gamma \cup \delta - P \times (0,1)$ and $N(a)$ be a regular neighborhood of a in $M^* - P \times (0,1)$ such that

1) each of $N(\delta) \cap W'$ and $N(\delta) \cap V'$ is a half 3-ball,

2) $N(a) \cap N(\gamma) \subset N(a)$ where $N(\gamma)$ is defined in Section 2.

Now let $V^* = V \cup E \times I$. Then $M^* = W \cup V^*$ is a Heegaard splitting of M^*. Let $P^* = \partial N(a) \cup P \times \{0,1\} - int(N(a) \cap (P \times \{0,1\}))$. Then P^* is a 2-sphere which intersects $S = \partial_+ W = \partial_+ V^*$ in an essential simple closed curve. Now by observations, $M^* = W \cup V^*$ is the connected sum of $M' = W' \cup V'$ and a genus one Heegaard splitting of $S^2 \times S^1$ along P^*.

Since $M' = W' \cup V'$ is unstabilized, by Lemma 3.1, $M^* = W \cup V^*$ is unstabilized. Note that $V^* = V \cup E \times I$. Hence $M = W \cup V$ is also unstabilized. Q.E.D.

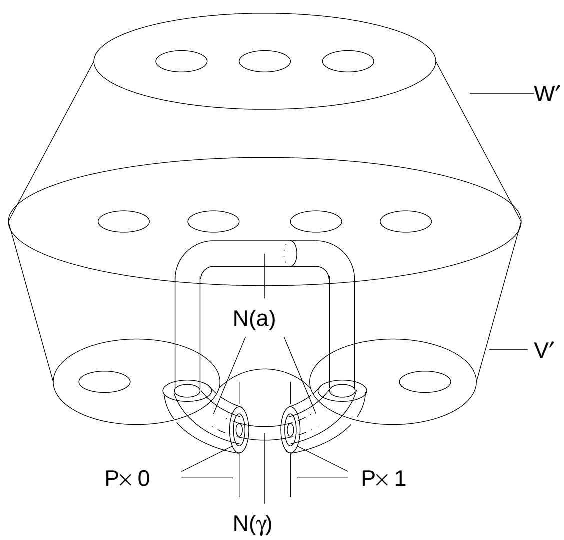

Figure 4

Lemma 3.2 and Lemma 3.3 mean that Heegaard genus is additive under boundary connected sums and self-boundary connected sums even if Heegaard genus is not minimal.

Lemma 3.4 ([CG]). Any Heegaard splitting of a ∂-reducible 3-manifold is ∂-reducible.

Theorem 1. Any Heegaard splitting of a ∂-reducible manifold M, say $M = W \cup_S V$, can be obtained by doing connected sums, boundary connected sums and self-boundary connected sums from Heegaard splittings of n manifolds $M_1, \ldots, M_n$ where M_i is either a solid torus or a ∂-irreducible manifold. Furthermore, $M = W \cup_S V$ is stabilized if and only if one of the factors is stabilized.

Proof. Suppose that $M = W \cup_S V$ is a Heegaard splitting of a ∂-reducible 3-manifold. If the genus of $M = W \cup_S V$ is one, then M is a solid torus and $M = W \cup_S V$ is a trivial Heegaard splitting of M. So we may assume that the genus of $M = W \cup_S V$ is at least two.

By Lemma 3.4, there is an essential disk D such that D intersects S in an essential simple closed curve in S. We may assume that $D \cap W$ is a disk and $D \cap V$ is an annulus. That means that $\partial D \subset \partial_- V$. Now there are three cases:

Case 1. D is separating in M.

Now $M - D \times (0, 1)$ contains two components M_1 and M_2, $D \cap W$ separates W into two compression bodies W_1 and W_2 and $D \cap V$ separates V into two components V_1 and V_2. We assume that $W_1, V_1 \subset M_1$ and $W_2, V_2 \subset M_2$. Let $N(D \cap W \times \{0\})$ be a regular neighborhood of $D \cap W \times \{0\}$ in W_1 and $N(D \cap W \times \{1\})$ be a regular neighborhood of $D \cap W \times \{1\}$ in W_2. Then $V_1 \cup N(D \cap W \times \{0\})$ and $V_2 \cup N(D \cap W \times \{1\})$ are two compression bodies. Hence $(W_1 - N(D \cap W \times \{0\})) \cup (V_1 \cup N(D \cap W \times \{0\})$ is a Heegaard splitting of M_1 and $(W_2 - N(D \cap W \times \{1\})) \cup (V_2 \cup N(D \cap W \times \{1\})$ is a Heegaard splitting of M_2. By definition, $W \cup V$ is a boundary connected sum of $(W_1 - N(D \cap W \times \{0\})) \cup (V_1 \cup N(D \cap W \times \{0\})$ and $(W_2 - N(D \cap W \times \{1\})) \cup (V_2 \cup N(D \cap W \times \{1\})$.

By Lemma 2.2, $W \cup V$ is stabilized if and only if one of $(W_1 - N(D \cap W \times \{0\})) \cup (V_1 \cup N(D \cap W \times \{0\})$ and $(W_2 - N(D \cap W \times \{1\})) \cup (V_2 \cup N(D \cap W \times \{1\})$ is stabilized.

Case 2. D is non-separating in M, but ∂D is separating in ∂M.

Claim 1. $D \cap W$ is non-separating in W.

Proof. Suppose, otherwise, that $D \cap W$ is separating in W. Then $\partial(D \cap W)$ is separating in $\partial_+ W = \partial_+ V$. Since V is a compression body, $D \cap V$ is separating in V. Hence D is separating in M, a contradiction. Q.E.D. (Claim 1)

Now $M - D \times (0,1)$ is a manifold $M^{'}$. Since $D \cap W$ is a non-separating disk in W, $W - (D \cap W) \times (0,1)$ is a compression body, say W^{*}. Let $N((D \cap W) \times \{0\})$ be a regular neighborhood of $(D \cap W) \times \{0\}$ and $N((D \cap W) \times \{1\})$ be a regular neighborhood of $(D \cap W) \times \{1\}$ in W^{*}. Then $(V - D \times (0,1)) \cup N((D \cap W) \times \{0\}) \cup N((D \cap W) \times \{1\})$ is a compression body, say $V^{'}$, in M'. Note that the closure of $W^{*} - (N((D \cap W) \times \{0\}) \cup N((D \cap W) \times \{1\}))$, say $W^{'}$, is also a compression body. By definition, $W \cup V$ is a self-boundary connected sum of $W^{'} \cup V^{'}$.

By Lemma 3.3, $W \cup V$ is stabilized if and only if $W^{'} \cup V^{'}$ is stabilized.

Case 3. D is non-separating in M, and ∂D is non-separating in ∂M.

Claim 2. $\partial(D \cap W)$ is non-separating in $S = \partial_{+}V = \partial_{+}W$.

Proof. Suppose, otherwise, that $\partial(D \cap W)$ is separating in S. Without loss of generality, we may assume that $\partial_{-}V$ contains only one component. Let V^{*} be the manifold obtained by attaching a handlebody H to V along $\partial_{-}V$ such that ∂D bounds a disk D^{*} in H. Then V^{*} is a handlebody and $(D \cap V) \cup D^{*}$ is a disk in V^{*}. Since $\partial(D \cap W)$ is separating in S, $(D \cap V) \cup D^{*}$ is separating in V^{*}, but D^{*} is non-separating in H, a contradiction. Q.E.D. (Claim 2)

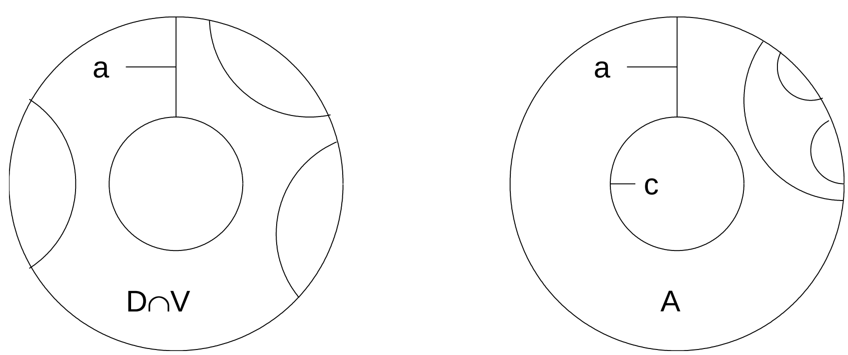

Figure 5

Claim 3. There is an annulus A such that

1) one component of A lies in $\partial_{+}V$ and the other lies in $\partial_{-}V$, and

2) A intersects the annulus $D \cap V$ in only one essential arc.

Proof. Now since $\partial_1(D\cap V)$ in $\partial_- V$ is a non-separating curve, there is a curve in $\partial_- V$, say c, such that $\mid \partial_1(D\cap V)\cap c \mid= 1$. Then c together with a simple closed curve in $\partial_+ V$ cobound an annulus, say A. We may assume that $\mid A\cap(D\cap V)\mid$ is minimal among all such annuli. Now we prove $\mid A\cap(D\cap V)\mid= 1$.

Note that A and $D\cap V$ are incompressible in V. Hence $A\cap(D\cap V)$ is a set of arcs. Since c intersects $\partial_1(D\cap V)$ in one point, there is only one arc, say a, in $A\cap(D\cap V)$ which is essential in both A and $D\cap V$. See Figure 5.

Suppose that $\mid A\cap(D\cap V)\mid> 1$. Let b be an arc in $A\cap(D\cap V)$ which is outermost in $D\cap V$, then it, with a sub-arc of $\partial_2(D\cap V)$, cobound a disk E in $D\cap V$ such that $intE$ is disjoint from A. Now b, with a sub-arc of $\partial_2 A$, cobound a disk $E^{'}$ in A. Thus $A^{'} = (A - E)\cup E^{'}$ is also an annulus, but $A^{'}$ can be isotoped so that $|A^{'}\cap(D\cap V)| < |A\cap(D\cap V)|$, a contradiction. Q.E.D. (Claim 3)

By Claim 3, there is an annulus A which intersects the annulus $D\cap V$ in only one arc. We may assume that $\partial D\subset F\subset\partial_- V$. Now Let $N = N(A\cup(D\cap V))$ and A^* be the closure of $\partial N(A\cup(D\cap V)) - \partial_- V\cup\partial_+ V$. Then A^* is also an annulus in V. We may assume that $\partial_1 A^*\subset\partial_+ V$ and $\partial_2 A^*\subset F$. Since the genus of $M = W\cup_S V$ is at least two, $\partial_1 A^*$ is an essential separating simple closed curve in $\partial_+ V$ which bounds a disk B in W. Now there are two subcases:

Case 3.1. F is a torus.

In this case, $\partial_2 A^*$ bounds a disk B^* in V. Now let $P = B\cup A^*\cup B^*$. Then P is a 2-sphere which intersects $\partial_+ V$ in an essential simple closed curve. That means that $M = W\cup V$ is a connected sum of two Heegaard splittings.

By Lemma 3.1, $W\cup V$ is stabilized if and only if one of the two factors is stabilized.

Case 3.2. $g(F)\geq 2$.

Now $\partial_2 A^*$ is an essential separating simple closed curve in $\partial_- V$. $A^*\cup B$ is an essential disk which intersects $\partial_+ V$ in an essential simple closed curve. By Case 1 and Case 2, $W\cup V$ is a boundary connected sum or a self-boundary connected sum of Heegaard splittings .

Now by induction on $g(\partial_- V)$ and $g(\partial_+ V)$, we can prove Theorem 1. Q.E.D

Corollary 2. Any unstabilized Heegaard splitting of a handlebody H is trivial.

Proof. We shall proof this corollary by induction on the genus of H. By [W], the unstabilized Heegaard splitting of a 3-ball is trivial.

Now suppose that H is a handlebody of genus at least 1, and $H = W \cup_F V$ is a unstabilized Heegaard splitting of H such that W is a handlebody, $\partial H = \partial_- V$ and $g(F) > g(\partial_- V)$. Since H is irreducible and ∂-reducible, by Lemma 3.4, there is an essential disk D such that D intersects F in an essential simple closed curve in F. We may assume that $D \cap W$ is a disk and $D \cap V$ is an annulus. Hence $\partial D \subset \partial_- V$.

If the genus of $\partial_- V$ is one, then by Case 3.1 in the proof of Theorem 1, $H = W \cup_F V$ is a connected sum of two Heegaard splittings, but H is irreducible and $g(F) > 1$, by [W], $W \cup V$ is stabilized, a contradiction. So it must be that $g(F) = g(\partial_- V)$, and the Heegaard splitting is trivial.

If the genus of $\partial_- V$ is larger then one, by Cases 1 and 2 in the proof of Theorem 1, $W \cup V$ is stabilized, a contradiction. So it must be that $g(F) = g(\partial_- V)$, and the Heegaard splitting is trivial. Q.E.D.

Another proof of Corollary 2 is given by Fengchun Lei from Scharlemann-Thompson's results.

References.

[CG]A. Casson and C. Gordon, Reducing Heegaard Splittings, Topology and its Applications 27(1987) 275-283.

[H]W. Haken, Some results on surfaces in 3-manifolds, Studies in Modern Topology (Math. Assoc. Amer., distributed by: prentice-Hall, 1968), 34-98.

[L]F. Lei, On stability of Heegaard splittings, Math. Proc. Cambridge Philos. Soc., 129(2000), 55-57.

[Q]R. Qiu, Stabilization of Reducible Heegaard Splittings, Preprint.

[ST] M. Scharlemann and A. Thompson, Heegaard splittings of (surface) x I are standard, Math. Ann. 295 (1993) 549-564.

[W] F.Waldhausen, Heegaard-Zerlegungen der 3-sphere, Topology 7 (1968)195-203.

Jiming Ma

Department of Mathematics

Jilin University

Changchun, 130023

China.

E-mail:majiming2000@sohu.com

Ruifeng Qiu

Department of Mathematics

Dalian University of Technology

Dalian, 116024

China.

qiurf@dlut.edu.cn